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AN APPLICATION OF A FUZZY RANDOM VARIABLE TO VULNERABILITY MODELING

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Fuzzy random variables have unique value in a modeling context because of their ability to distinguish model components which are incompletely known due to stochastic variation from components which are unknown due to imprecise measurement or inherent vagueness in their quantification. In this paper we present a straightforward application of fuzzy methods to an important and ongoing defense problem, that of evaluating the susceptibility of an armored vehicle to attack.

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1. Introduction

Kwakernaak, in a seminal paper [6], introduced the notion of a fuzzy random variable as a random variable whose values are not real but fuzzy numbers. Expectation and probabilities relating to a fuzzy random variable are developed as images of a fuzzy set, representing the fuzzy random variable, under appropriate mappings. Kwakernaak's constructions, or slight variations of them, have received theoretical elaboration, primarily directed toward the extension of classical probability laws. For example, Kruse [5] and Miyakoshi and Shimbo [8] report on a strong law of large numbers. Boswell and Taylor [2] provide an analogue of the central limit theorem for fuzzy random variables admitting a moment generating function, while Puri and Ralescu [11] outline a theory similar to Kwakernaak's and derive a dominated convergence theorem. Stein and Talati [14], following Nahmias [9], develop a theory specifically for convex fuzzy random variables.

Fuzzy random variables have unique value in a modeling context because of their ability to distinguish model components which are incompletely known due to stochastic variation from components which are unknown due to imprecise measurement or inherent vagueness in their quantification. In conventional practice the latter type of unknown is often treated as a crisp value in a sensitivity study, or represented as a randomly varying quantity, and in either case misconstrued. By contrast, fuzzy sets offer a conceptually faithful representation of the nature of vague unknowns, and the rules of fuzzy logic supply an explicit framework for manipulating and composing any number of vague quantities. When combined with the treatment of stochastic quantities, as in fuzzy random variables, complex modeling problems may be addressed. Moreover, the fuzzy sets obtained as end products of the modeling process immediately summarize the amount and type of uncertainty remaining in computed values, and so inform the interpretation and application of model conclusions.

In section 3 we present a straightforward application of fuzzy methods to an important and ongoing defense problem, that of evaluating the susceptibility of an armored vehicle to attack. The presence of vague information in this type of vulnerability modeling has been cited by Schlegel, et. al. [13], who suggest fuzzy sets as a modeling tool. Section 3 begins with an overview of current approaches to vulnerability assessment, indicating where fuzzy sets may be appropriate in the modeling process. For illustration, an implementation using fuzzy random variables is presented which achieves general applicability by utilizing an existing knowledge base and existing computer models with only incidental revisions. Necessary theoretical background is covered in section 2, and a brief discussion follows in section 4.

2. Fuzzy Random Variables

Kwakernaak [6,7] defines a fuzzy set $\mathbf{f}$ as a triple $\mathbf{f} = (A, t, p)$ consisting of a basic set A, a logical proposition p which can be applied to every member of the basic set, and a function t which assigns to every member $x \in A$ a truth value $t(p(x))$ indicating the appropriateness of the proposition p as applied to x. Most authors suppress the proposition p notation, since it is implicit in the organizing principle of the fuzzy set, and compose the proposition and truth value into a membership function $\mu: A \to [0, 1]$ which acts on the basic set, $\mu(x) = t(p(x))$. Thus $\mathbf{f}$ would be written $\mathbf{f} = (A, \mu)$; we shall adopt this convention.

An α-level set corresponding to a given fuzzy set $\mathbf{f} = (A, \mu)$ is an ordinary non-fuzzy set, denoted

$$L_\alpha(\mathbf{f}) = \{x \,\varepsilon\, A \mid \mu(x) \geq \alpha\}. \tag{2.1}$$

A fuzzy number is a fuzzy set having the real line **R** as its basic set. The fuzzy number **f**, or its membership function μ, is said to be unimodal if for every $\alpha \,\varepsilon\, (0, 1]$, $L_\alpha(\mathbf{f})$ is convex.

Fuzzy random variables are constructed as a means of modeling phenomena which could properly be described by ordinary real random variables defined on a probability space (Ω, F, P), but which are partially obscured by fuzzy perception of the real line. In particular, if U_0 is the underlying random variable and ω is the outcome of a random experiment, the exact value $U_0(\omega)$ is unobservable; instead, it is assumed that a fuzzy number $\mathbf{f} = (\mathbf{R}, X_\omega)$ is known which characterizes the result $U_0(\omega)$. The mapping $X:\Omega \rightarrow S$ from the sample space Ω into the class of admissible membership functions S, given by $X(\omega) = X_\omega$ supplies a membership function for each random outcome, and is called a (fuzzy) perception function. To the observer who must perceive random outcomes via X, the identity of U_0 is lost, and there may be many reconstructions of U_0 which are amenable to fuzzy perception. By the standard operations of fuzzy logic [4], X generates a valuation function which applies to random variables as entities. Namely, if U is an F-measurable random variable, then

$$\mu_X(U) = \inf_{\omega \,\varepsilon\, \Omega} X_\omega(U(\omega)) \tag{2.2}$$

is the valuation of its suitability as a reconstruction of U_0.

To ensure proper measurability relationships in the definition of a fuzzy random variable, it is necessary to impose some structure on the perception function, X, and on the basic set from which candidates for reconstruction of U_0 are drawn. The reader is referred to Kwakernaak [6] for a detailed exposition of structural requirements, and to [2] for some simplifying modifications. Briefly, we admit as a basic set U_F, the set of all F-measurable random variables on Ω, and enforce partial retention of the structure of (Ω, F, P) through the requirement that for all $\alpha \,\varepsilon\, (0, 1]$ the functions

$$U_\alpha^{*}(\omega) = \inf \{x \,\varepsilon\, \mathbf{R} \mid X_\omega(x) \geq \alpha\} \tag{2.3}$$

and

$$U_\alpha^{**}(\omega) = \sup \{x \,\varepsilon\, \mathbf{R} \mid X_\omega(x) \geq \alpha\}$$

are measurable with respect to (Ω, F). The sigma algebra generated by the random variables U_α^{*}, $\alpha \,\varepsilon\, (0, 1]$ and U_α^{**}, $\alpha \,\varepsilon\, (0, 1]$ will be denoted by $\sigma(X)$; the set of all $\sigma(X)$-measurable random variables on Ω will be denoted by χ.

Letting U_F be the collection of all F-measurable random variables on Ω, the *fuzzy random variable* induced by X is defined to be the fuzzy set

$$\mathbf{X} = (U_F, \mu_X) .$$

Some properties of a fuzzy random variable may be obtained directly by the extension principle [14]. For example, the expectation of a fuzzy random variable $\mathbf{X}$ is a fuzzy number

$$E\mathbf{X} = (\mathbf{R}, \mu_{E\mathbf{X}})$$

with membership function given by

$$\mu_{E\mathbf{X}}(x) = \sup_{U \varepsilon U_F :\, EU = x} \; \inf_{\omega \varepsilon \Omega} \; X_\omega (U(\omega))$$

$$= \sup_{U \varepsilon U_F :\, EU = x} \mu_X (U), \quad x \varepsilon \mathbf{R}. \tag{2.4}$$

In (2.4), E denotes the usual mathematical expectation.

A fuzzy random variable $\mathbf{X}$ is called unimodal if for each $\omega \varepsilon \Omega$, the membership function X_ω is unimodal. Kwakernaak [6] shows that if $\mathbf{X}$ is unimodal the basic set U_F may be restricted to χ, the set of all $\sigma(X)$-measurable random variables on Ω:

Theorem (2.1). If $\mathbf{X}$ is unimodal, then

$$\mu_{E\mathbf{X}}(x) = \sup_{U \varepsilon \chi :\, EU = x} \; \inf_{\omega \varepsilon \Omega} \; X_\omega(U(\omega)), \quad x \varepsilon \mathbf{R}. \tag{2.5}$$

3. Vulnerability Modeling

Succinctly, vulnerability modeling is an attempt to characterize the interaction between a target (armored vehicle, aircraft, bunker, ...) and a munition (kinetic energy penetrator, shaped charge, explosive device, ...), and to assess quantitatively the damage related to the target-munition encounter. Our focus here will be upon evaluating the susceptibility of an armored vehicle to attack by a kinetic energy penetrator, and a class of vulnerability models appropriate to this situation known as point-burst models. TANKILL, a model developed in the United Kingdom, the VAST model, developed in the United States, the APAS model, developed in Sweden, and the PVM model, developed in West Germany, are all examples of modern point-burst vulnerability models [12].

Typically, perforation of armor by a projectile (penetrator) will generate a spray of armor fragments inside the vehicle which, along with a portion of the projectile itself, may strike a number of internal components. The tasks for vulnerability assessment are to characterize the fragment spray which follows a munitions impact, to determine which components of the vehicle are struck, to evaluate how these components are affected individually, and to summarize the consequences for the vehicle as a whole.

A vulnerability model will include one functional unit which employs mathematical sub-models to simulate the physical phenomena governing armor penetration and fragment formation. The sub-models themselves are complex entities, relying upon a variety of assumptions, generally with limited experimental support. Thus they may be considered as potential candidates for fuzzy extensions, as indeed may many aspects of vulnerability assessment. However, for present purposes we simply accept that the vulnerability model has at its disposal a characterization of the fragment spray resulting from any given munitions impact. Also there will be a module supplying a geometric description of the vehicle, including the layout of internal components, and such details of mechanical structure, shielding, and hydraulics as are known and considered important.

Point-burst models simulate the trajectories of the fragments with a bundle of rays emanating from the point where the penetrator (shotline) erupts through the interior surface of the armor plate. Each ray is traced explicitly through the computer description of the target vehicle to determine which components are likely to be struck (Figure 1). Interior components may be struck by the shotline or by the fragment rays; in either case, pertinent geometric data are recorded, including lists of components struck, impact obliquities, and distances traveled between and through components. From such information the damage to individual components must be assessed.

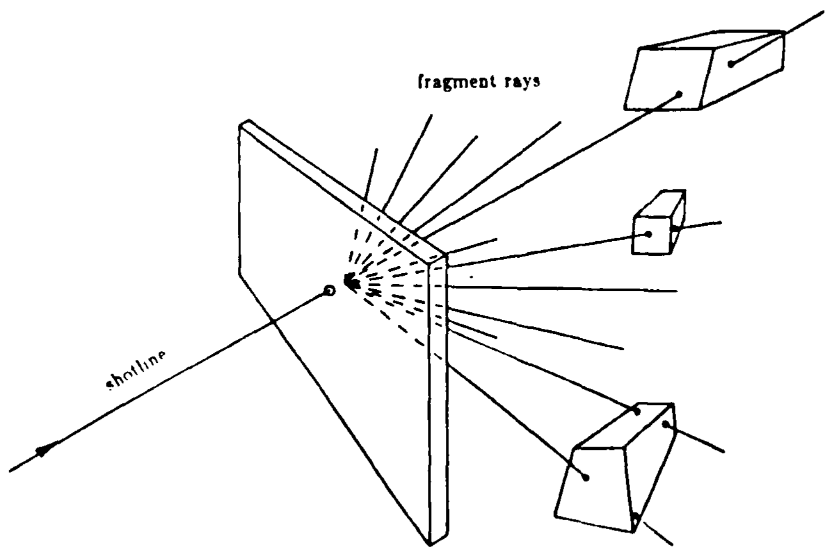

Fig. 1. Ray tracing behind armor.

Predicting the effects of fragment impacts on individual components is one of the most important aspects of the point-burst vulnerability model, and one of the most troublesome. The effect of one impact may be inconsequential, or it may functionally destroy the component. The typical point-burst model includes a module for estimating the probability that a component will be destroyed, given that it is struck by a fragment. Often the only damage mechanism which is overtly considered is perforation; usually ignored are such possibilities as shock and blast effects, significant nondestructive damage, and the cumulative effect of impacts from multiple fragments.

Another key element of the point-burst method is the linkage which relates component damage to the degradation of total vehicle performance. The procedure for assessing consequences of aggregate component damage varies among the different models, but generally it involves probability theory, heuristics, and engineering judgement, in some combination. We will limit ourselves here to one aggregate damage measure, called the overall vehicle probability-of-kill, P_k, which we will describe as it is implemented in the VAST model.

For a specific direction of attack, a selection of impact locations on the target is determined by superimposing a rectangular grid (Figure 2) and choosing a single point of impact within each grid cell. Then the VAST model, using the point-burst approach, provides a corresponding probability-of-kill estimate for the vehicle given a hit in each cell i, denoted by $\hat{P}_{k|h_i}$. If an aim-point on the vehicle is designated, then eventual impact will occur in various cells of the grid with relative frequencies given by a bivariate probability density located at the aim-point. Taking the corresponding weighted average of the cell $\hat{P}_{k|h_i}$'s, one arrives at an overall estimate of the probability-of-kill, $\hat{P}_k$, for the vehicle:

$$\hat{P}_k = \sum w_i \hat{P}_{k|h_i}. \tag{3.1}$$

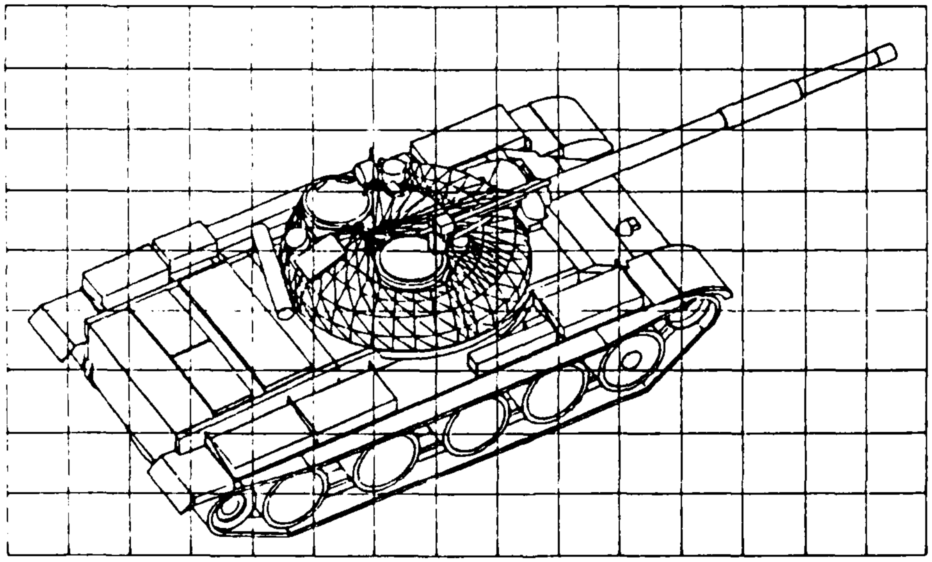

Fig. 2. Target description with superimposed rectangular grid.

Figure 3 presents an illustrative vulnerability assessment of an encounter between an armored vehicle and a kinetic energy penetrator. Each grid cell contains the conditional probability estimate that the tank will be killed should it sustain an impact in that cell. For the rectangle featured in Figure 3, the point-burst model provides an estimated probability-of-kill given a hit in that cell, $\hat{P}_{k|h_i} = 0.19$.

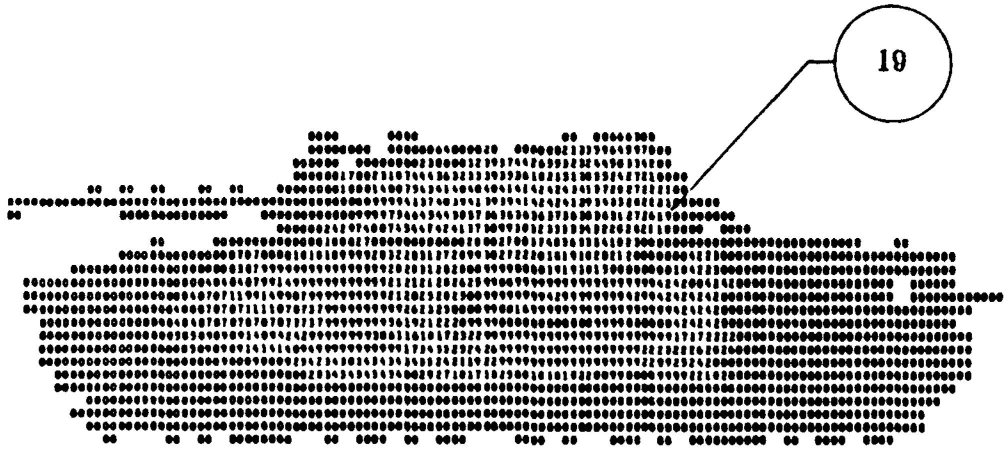

Fig. 3. Data summary of an encounter between an armored vehicle and a kinetic energy penetrator.

The value $\hat{P}_{k|h_i}$ represents a summary valuation of a complicated causal chain relating munitions impact to vehicle functionality. As we have previously indicated, the quantitative description of that causal chain is often arbitrary or incomplete. Experimental testing required to provide data for estimation and model validation is destructive, and the data base upon which these models are built may be modest, or in the case of conceptual systems, nonexistent. In addition, while certain damage-related measurements (velocity of impact, depth of penetration, component function, ...) may be determined within measurement error, many others (structural deformation, fracturing, ...) may not.

There is a stochastic component already implemented in existing vulnerability models, reflecting variation in outcome across replicate firings at the vehicle. Were the causal relationships involved in the estimation of $P_{k|h_i}$ completely specified, the uncertainty remaining in $\hat{P}_{k|h_i}$ could properly be represented by a confidence interval, obtained by analysis of firing-to-firing variation. In applications of interest to us, however, fuzzy unknowns inherent in the geometry of the vehicle and in the causal model are essential contributors to the total uncertainty in $\hat{P}_{k|h_i}$. Thus we have considered the introduction of fuzzy set methods to the existing body of vulnerability software.

Certainly it is possible to use fuzzy sets to describe unknowns at any level of aggregation in the causal chain, and then to compose the chain of effects via fuzzy random variables and the operations of fuzzy logic. We have chosen to introduce fuzzy sets at a relatively high level of aggregation, namely in the representation of the estimated cell probabilities, $\hat{P}_{k|h_i}$. The reasons for this choice are two-fold. First, and most importantly, the impact cell is the level of aggregation at which experienced modelers are most comfortable and best informed. (A primary criterion for fuzzy set methodology should be its ability to incorporate heuristic judgement, in a manner appropriate to the problem at hand, as directed by the same heuristic judgement). Second, there is a practical advantage in retaining, as a baseline, the meticulously developed models and software which underlie current estimates of $P_{k|h_i}$. Then, by adopting fuzzy sets of suitable form to characterize the precision of cell probabilities, the entire "fuzzification" process may be economically automated.

A particularly convenient numerical implementation results if we replace each cell $\hat{P}_{k|h_i}$ with a fuzzy number whose membership function is illustrated in Figure 4. The width of the interval on which $\mu(x)$ assumes its maximum value is chosen to be $\sigma^2 = P_{k|h_i}(1 - P_{k|h_i})$, the variance of the Bernoulli distribution modeling the individual cell probabilities. The introduction of the variance into the membership function is made to reflect the fact that larger (smaller) variance corresponds to greater (less) uncertainty about the value of $P_{k|h_i}$.

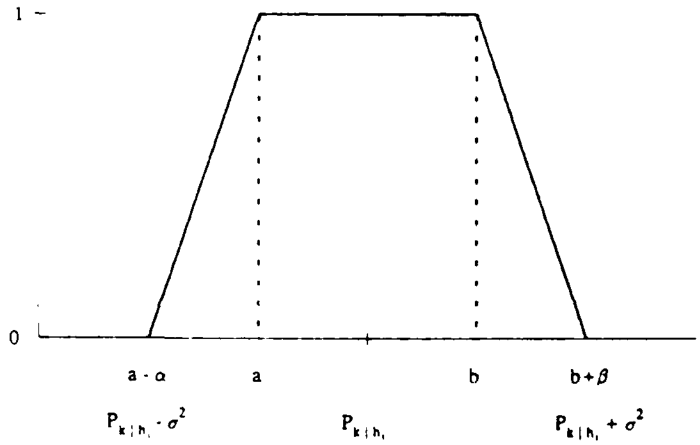

Fig. 4. Membership function for fuzzy $P_{k|h_i}$.

This procedure for constructing the fuzzy set is of course arbitrary, but qualitatively in accord with common methods of evaluating Bernoulli probabilities. One may easily entertain alternate formulations. Allowing for random variation in the site of munition impact, the correspondence between impact location and the now fuzzy probability-of-kill defines a fuzzy perception function, $X(\omega) = X_\omega$. The vulnerability calculation (3.1) carried out on fuzzy $P_{k|h_i}$ is equivalent to computing the expected value of the discrete fuzzy random variable X induced by the perception function X.

Since the membership functions in Figure 4 are unimodal, the expectation of X is a fuzzy number EX with membership function given by

$$\mu_{EX}(x) = \sup_{U \in \chi:\, EU = x} \; \inf_{\omega \in \Omega} X_\omega(U(\omega)), \quad x \in \mathbf{R}. \tag{3.2}$$

This expression can be evaluated using the α-level sets (2.1). Given the family of level sets $L_\alpha(\cdot)$, the membership function $\mu_{EX}(x)$ may be recovered with the aid of the formula

$$\mu(x) = \sup \{\alpha \varepsilon [0, 1] | x \varepsilon L_\alpha(\cdot)\}, \quad x \varepsilon \mathbf{R}. \tag{3.3}$$

For the membership functions shown in Figure 4, this computation can be simplified using procedures given by Dubois and Prade [3], or Bonnisone [1]. They show that the membership function may be represented as a four-tuple (a, b, α, β) and that fuzzy arithmetic may be carried out on fuzzy numbers of this type simply by operating on the coordinates of the four-tuple. The relations

$$\tilde{m} + \tilde{n} = (a+c, b+d, \alpha+\gamma, \beta+\delta) \tag{3.4}$$

and

$$\tilde{m} \times \tilde{n} = (ac, bd, a\gamma + c\alpha - \alpha\gamma, b\delta + d\beta + \beta\delta)$$

for addition and multiplication of the fuzzy numbers $\tilde{m} = (a, b, \alpha, \beta)$ and $\tilde{n} = (c, d, \gamma, \delta)$ are sufficient for our purpose of fuzzifying the vulnerability calculation (3.1). For an arbitrary membership function, an iterative algorithm on the α-level sets (3.3), detailed in Kwakernaak [7] may be required.

Applying relations (3.4) to the data in Figure 3, we obtain for EX the membership function shown in Figure 5. The interpretation of the resultant $\mu_{EX}(x)$ is that estimates of overall P_k in the interval [.18, .23] are, within our framework of uncertainty, wholly plausible. The VAST model has been criticized for producing point estimates $\hat{P}_k$, without an accompanying interval estimate to gauge their accuracy. We offer one caution to the reader: the impulse to take the level set $L_{.90}(\cdot)$, say, and treat it formally as a 90% confidence interval for P_k must be resisted; there are no probability statements carried by the α-level sets.

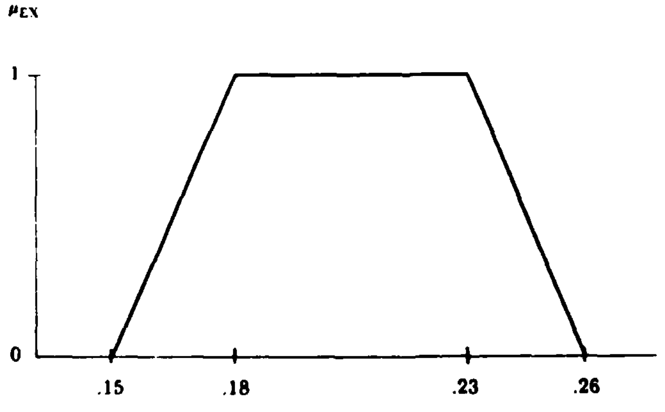

Fig. 5. Membership function for expectation EX of fuzzy random variable X.

4. Conclusion

We have modeled the effect of uncertainty in the cell $P_{k\,|\,h_i}$ estimates on the overall probability-of-kill estimate $\hat{P}_k$ in a direct way, and distinguished between randomness and uncertainty in the point-burst vulnerability model. Moreover, the framework is in place to consider $P_{k\,|\,h_i}$ membership functions far more intricate than the one shown in Figure 4.

A value such as $\hat{P}_k$ seldom remains in isolation, and finds use in a larger strategic context. The $\hat{P}_k$ value may serve as a parameter in large-scale war games, and is of value to armament designers. Using fuzzy random variables and the rules of fuzzy logic, information from diverse sources and with varying degrees of quantification may be coordinated. A decision-maker in such a context then obtains a valuation of the combined strength of evidence for a given decision, along with a diagnostic assessment of the evidence upon which that decision is based. This is a significant methodological improvement.

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List of Symbols

EX	fuzzy expectation of a fuzzy random variable **X**
f	a fuzzy set
$L_\alpha(\mathbf{f})$	an alpha-level set of a fuzzy set **f**
$\tilde{m}$	a fuzzy number
$\tilde{n}$	a fuzzy number
P_k	probability-of-kill
$\hat{P}_k$	estimated probability-of-kill
$\hat{P}_{k\mid h_i}$	estimated conditional probability-of-kill
R	the real line
S	a class of admissible membership functions
U_0	an ordinary random variable on Ω
$U_{F_\bullet}$	the set of all F-measurable random variables on Ω
U_α^{*}	a measurable function
U_α^{**}	a measurable function
X	a fuzzy perception function
X	a fuzzy random variable induced by X
X_ω	a membership function corresponding to a random outcome ω
χ	the set of all $\sigma(X)$-measurable random variables on Ω
μ	a membership function for a fuzzy set
μ_x	a membership function for F-measurable random variable U
(Ω, F, P)	a general probability space
$\sigma(X)$	sigma algebra generated by U_α^{*} and U_α^{**}

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